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A Comparative Study of the Group Runs and Side Sensitive Group Runs Control Charts

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ABSTRACT

The Group Runs (GR) and Side Sensitive Group Runs (SSGR) control charts are the improvement of the synthetic chart without and with side sensitivity. Both the SSGR and GR charts are effective for detecting small to moderate process mean shifts. The performances of the SSGR and GR charts, in terms of the average time to signal (ATS), are compared in this paper. In this comparative study, the Monte Carlo simulation approach by means of the Statistical Analysis System (SAS) software is used to compute the ATS values for the GR and SSGR charts. The results revealed that the SSGR chart's performance is better than that of the GR chart for all levels of mean shifts.

Keywords: Average time to signal, GR chart, SSGR chart

INTRODUCTION

The Shewhart's $\bar{X}$ chart has wide applications in the service and manufacturing sectors. The Shewhart's $\bar{X}$ chart is effective for detecting large process mean shifts. On the other hand, both the Exponentially Weighted Moving Average (EWMA) and Cumulative Sum (CUSUM) control charts, introduced by Roberts (1959) and Page (1954), respectively, are used to detect small process mean shifts. Recently, many researchers have contributed to a wide variety of control charts to improve process monitoring, such as Costantino *et al*. (2015), Moraes *et al*. (2014), Ali and Riaz (2014), Haq *et al*. (2015), Chong *et al*. (2014), Teoh *et al*. (2014), Costa and Machado (2015) and many more.

The Conforming Run Length (CRL) chart was proposed by Bourke in 1991. Bourke (1991) referred to CRL as the number of inspected units between two consecutive defective units. When CRL < L_{CRL}, the CRL chart will indicate a shift in the process. Here, L_{CRL} is the lower limit of the CRL chart.

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The synthetic chart suggested by Wu and Spedding (2000) combines the CRL chart (CRL/S sub-chart) and the Shewhart $\bar{X}$ chart ($\bar{X}/S$ sub-chart). The sample (group) of the CRL/S sub-chart is considered as a unit. This synthetic chart is sensitive to small and moderate process mean shifts. Let μ_0 be the target value of the mean, σ be the standard deviation, k be the multiplier controlling the width of the control limits of the $\bar{X}/S$ sub-chart and n be the sample size. If a group mean is $\bar{X} \notin \left(\mu_0 - k\sigma/\sqrt{n},\ \mu_0 + k\sigma/\sqrt{n}\right)$, the group in the synthetic control chart is declared as non-conforming. The synthetic chart signals an out-of-control status when CRL $\leq L_S$ for the first time. Here, L_S is the lower limit of the CRL/S sub-chart. Davis and Woodall (2002) extended the idea of Wu and Spedding (2000) to propose a side sensitive version of the synthetic control chart. They stated that the synthetic chart with side sensitivity outperforms the synthetic chart. This synthetic chart with side sensitivity has the feature of the run rules chart with a head start. For the synthetic chart with side sensitivity, if two out of $(L_S + 1)$ group means lie outside of the $\bar{X}/S$ sub-chart's limits on the same side of the centre line, the process is declared as being out-of-control.

The Group Runs (GR) control chart proposed by Gadre and Rattihalli (2004) is a combination of an extended version of the CRL chart and the Shewhart $\bar{X}$ chart. Note that this GR chart is an improvement of the synthetic chart. Let L_g be the lower limit of the extended version of the CRL chart, which is a sub-chart of the GR chart.

For the GR chart, a process is declared as being out-of-control when two successive group-based CRLs $\leq L_g$ or the first group-based CRL $\leq L_g$ for the first time. Note that both the non-conforming groups in the two successive CRL values either have shifts on the same or opposite side of μ_0. Besides that, Gadre and Rattihalli (2007) developed a Side Sensitive Group Runs (SSGR) control chart, which is an extension of the group runs chart with side sensitivity. In the SSGR chart, the process is declared as being out-of-control if two successive values CRL $\leq L_{ssg}$ or the first group-based CRL $\leq L_{ssg}$ for the first time. Note that both the non-conforming groups for the two successive CRL values have shifts on the same side of μ_0. Here, L_{ssg} denotes the lower limit of the CRL chart, which is a sub-chart of the SSGR chart. Moreover, Gadre *et al*. (2010) modified the SSGR chart to develop the Side Sensitive Modified Group Runs (SSMGR) chart for detecting shifts in the process mean. Gadre (2014) also extended the univariate GR chart to bivariate and multivariate GR charts for monitoring process variability. The GR and SSGR charts can be used in any process monitoring situations in manufacturing, service industries, healthcare etc. The GR and SSGR charts are found to be better than the EWMA and CUSUM charts in detecting moderate and large shifts.

Gadre and Rattihalli (2007) compared the performance of the SSGR chart with the GR, Shewhart's $\bar{X}$ and synthetic charts. However, in the comparison, the sample size (n) was not fixed, but was chosen to minimise the average time to signal a shift. This resulted in a very large n. For example, in one of the examples, the optimal n was 186, 102, 98 and 89 for the Shewhart's $\bar{X}$, synthetic GR and SSGR charts, respectively. It may not be feasible for practitioners to adopt such a large sample size. Thus, for this paper, we allowed the practitioner to fix the sample size at a particular value, then selected the optimal control limits which would minimise the average time to signal a shift. We focused our attention on small values of n, as small sample sizes are preferred in industrial applications.

The performances, in terms of the average time to signal (ATS), of the SSGR and GR charts for detecting process mean shifts are compared in this paper. The objective of this study was to determine which control chart gives a better performance. The remainder of this paper is organised as follows: Sections 2 and 3 explain the design of the GR and SSGR charts, respectively. Section 4 studies and compares the performance of the GR and SSGR charts. Conclusions are drawn in Section 5.

DESIGN OF THE GR CONTROL CHART

The GR chart operates by declaring a sample of n items as being non-confirming when the sample mean falls outside the lower $\left(L_{\bar{X}|S}\right)$ or upper $\left(U_{\bar{X}|S}\right)$ control limits of the $\bar{X}$ sub-chart. The number of conforming samples between two successive non-conforming samples is defined as the CRL. The process is declared as being out-of-control when the number of two successive CRLs $\leq L_g$ or the first CRL $\leq L_g$ for the first time. Readers may refer to Gadre and Rattihalli (2004) for a detailed explanation of the GR chart.

The average time to signal a shift of size δ, ATS(δ) is given as (Gadre & Rattihalli, 2004):

$$\text{ATS}(\delta) = \frac{n}{P(\delta)} \frac{1}{\left[1 - \left\{1 - P(\delta)\right\}^{L_g}\right]^2}, \tag{1}$$

where $P(\delta)$ is the probability of the occurrence of a non-conforming sample for a shift δ and is given as:

$$\begin{aligned} P(\delta) &= 1 - P\left(L_{\bar{X}|S} < \bar{X} < U_{\bar{X}|S} \,\middle|\, \bar{X} \sim N\left(\mu_0 + \delta\sigma, \frac{\sigma}{\sqrt{n}}\right)\right) \\ &= 1 - \Phi\left(k - \delta\sqrt{n}\right) + \Phi\left(-k - \delta\sqrt{n}\right), \end{aligned} \tag{2}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function. The derivation of Equation (1) is shown in Gadre and Rattihalli (2004).

To find the optimal chart parameters (k, L_g) for a given group size, the optimisation algorithm for the GR chart is taken as being similar to that of the synthetic control chart, as proposed by Wu and Spedding (2000). The optimal chart parameters are the parameters that minimise the ATS(δ) subject to the chosen in-control ATS (ATS_0) constraint, where $\text{ATS}_0 = n \text{ x } \text{ARL}_0$. The procedure to obtain the optimal chart parameters is outlined as follows:

1. Specify the values of n, μ_0, σ, δ and ARL_0.
2. Let $L_g = 1$.
3. Numerically solve Equation (1) for k when ATS(0) = ATS_0, where $\text{ATS}_0 = n \times \text{ARL}_0$ Parameters (L_g , k) are candidates for an optimal GR chart, since they produce an in-control ATS value of ATS_0.
4. Calculate the out-of-control ATS, ATS(δ) by substituting the current values (L_g,k) into Equation (1).
5. If the out-of-control ATS is reduced to a given precision, let $L_g = L_g + 1$ and go back to Step 3. Otherwise, proceed to Step 6.

6. Take the immediate previous values of (L_g, k) as the optimal control limits of the GR chart.

In this paper, the optimal parameters are computed using Mathematica by implementing Steps 1 to 6.

DESIGN OF THE SSGR CONTROL CHART

The SSGR chart operates by declaring a sample of n items as non-confirming when the sample mean falls outside the lower $\left(L_{\bar{X}|S}\right)$ or upper $\left(U_{\bar{X}|S}\right)$ control limits of the $\bar{X}$ sub-chart. The process is declared as out-of-control when the number of two successive CRLs $\leq L_g$ or the first CRL $\leq L_g$ for the first time. However, unlike the GR chart, the two successive $\bar{X}$ samples corresponding to the two successive CRLs must fall on the same side of the target value μ_0. Readers may refer to Gadre and Rattihalli (2007) for a detailed explanation of the SSGR chart.

The ATS(δ) of the SSGR chart (Gadre and Rattihalli, 2007) is given as:

$$\text{ATS}(\delta) = \frac{n}{P(\delta)} \frac{1-\alpha(1-\alpha)A^2}{A^2[1+\alpha(1-\alpha)(A-2)]} \qquad (3)$$

where $P(\delta) = 1 - \Phi\left(k - \delta\sqrt{n}\right) + \Phi\left(-k - \delta\sqrt{n}\right)$,

$$A = 1 - \left[1 - P(\delta)\right]^{L_{ssg}},$$

and $$\alpha = \frac{\left[1 - \phi\left(k - \delta\sqrt{n}\right)\right]}{P(\delta)},$$

where Φ (·) is the standard normal probability density function. The derivation of Equation (3) can be found in Gadre and Rattihalli (2007).

We are interested to search for the optimal chart parameters (k, L_{ssg}) for a given sample size, n. The computation of the optimal chart parameters of the SSGR chart is similar to that of the GR chart described in the previous section but by replacing the notation L_g with L_{ssg}, Equation (1) with Equation (3) and the words ‘GR chart’ with ‘SSGR chart’.

A COMPARISON BETWEEN THE GR AND SSGR CHARTS BASED ON ATS

A comparison between the GR and SSGR charts, in terms of the ATS, is discussed in this section. The Statistical Analysis System (SAS) software was used to calculate the ATS via the simulation methods. The procedure to compute the ATS for the GR chart is outlined as follows:

1. Specify the values of n, μ_0, σ, δ, k and L_g.
2. Compute the lower and upper control limits (LCL and UCL) of the $\bar{X}$ sub-chart.
3. Set CRL = 0 and p = 0.
4. Set i = 1.

5. For j = 1 to n, generate X from a normal distribution with mean $\mu 0 + \delta\sigma$ and variance σ^2, then take the sample average as $\bar{X} = \frac{\sum_{j=1}^{n} X_j}{n}$. If LCL < $\bar{X}$ < UCL, go to Step 6; otherwise, go to Step 7.
6. Set a = 1, CRL = CRL + 1 and $i = i + 1$, then return to Step 5.
7. Set $p = p + 1$. If a = 1, set CRL = CRL + 1 and CRLL (p) = CRL; else if $a \neq 1$, set CRLL (p) = CRL. If p = 1, go to Step 8; otherwise, if $p \geq 3$, go to Step 9.
8. If CRLL (p) $\leq L_g$, ATS = $i \times n$; otherwise, i = i + 1 and return to Step 5.
9. If CRLL (p-1) $\leq L_g$ and CRLL (p)$\leq L_g$, ATS = $i \times n$; otherwise, $i = i + 1$ and return to Step 5.

Repeat Steps 3 to 9 for 10,000 times. The ATS is the average value of all the ATS values computed from the 10,000 simulation trials.

The ATS of the SSGR chart was obtained in similar manner (except that the side sensitivity feature was considered):

1. Specify the values of n, μ_0, σ, δ, k and L_{ssg}.
2. Compute the lower and upper control limits (LCL and UCL) of the $\bar{X}$ sub-chart.
3. Set CRLU = 0, CRLL = 0 and p = 0.
4. Set i = 1.
5. For j = 1 to n, generate X from a normal distribution with mean $\mu_{0+}\sigma\delta$ and variance σ^2 then take the sample average as $\bar{X} = \frac{\sum_{j=1}^{n} X_j}{n}$. If $LCL < \bar{X} < UCL$, go to Step 6; otherwise, go to Step 7.
6. Set a = 1, CRLU = CRLU + 1, CRLL = CRLL + 1 and $i = i + 1$, then return to Step 5.
7. Set $p = p + 1$. If $\bar{X} > UCL$, go to Step 8, while if $\bar{X} > LCL$, go to Step 9.
8. If a = 1, set CRLU = CRLU + 1 and CRLUU (p) = CRLU; otherwise, if $a \neq 1$, set CRLUU (p) = CRLU. If p = 1, go to Step 10; otherwise, if $p \geq 3$, go to Step 11.
9. 9. If a = 1, set CRLL = CRLL + 1 and CRLLL (p) = CRLL; otherwise, if $a \neq 1$, set CRLLL (p) = CRLL. If p = 1, go to Step 12; otherwise, if $p \geq 3$, go to Step 13.
10. If CRLUU (p) $\leq L_{ssg}$, ATS = $i \times n$; otherwise, $i = i + 1$ and return to Step 5.
11. If CRLUU (p -1) $\leq L_{ssg}$ and CRLUU (p) $\leq L_{ssg}$, ATS = $i \times n$; otherwise, $i = i + 1$ and return to Step 5.
12. If CRLLL (p) $\leq L_{ssg}$, ATS = $i \times n$; otherwise, $i = i + 1$ and return to Step 5.
13. If CRLLL (p - 1) $\leq L_{ssg}$ and CRLLL (p)$\leq L_{ssg}$, ATS = $i \times n$; otherwise, $i = i + 1$ and return to Step 5.

Repeat Steps 3 to 13 for 10,000 times. The ATS is the average value of all the ATS values computed from the 10,000 simulation trials.

To compare the GR and SSGR charts, the following combinations of input parameters were selected:

n :	3	5	7
δ_{opt} :	0.2	0.5	1.0
ARL_0 :	370	500	

Here, δ_{opt} denotes the size of a mean shift for which a quick detection is desired and ARL_0 is the in-control average run length. All the 18 possible combinations of the input parameters (n, δ_{opt}, ARL_0) are considered in this paper. These combinations of input parameters are selected so that practitioners can study the performances of the GR and SSGR charts for small sample sizes and small shift sizes. Small sample sizes are preferred in industrial applications, while the performance of the chart in detecting small shift sizes is important as large shift sizes are usually easily detected. Besides that, two different constraints in ARL_0 are used so that the effects of the constraints on the ATS can be studied. Practitioners can also choose other combinations of the input parameters according to their needs. The process is then monitored.

These input parameters (n, δ_{opt}, ARL_0) were used to obtain the optimal values, (k, L_g) and (k, L_{ssg}) of the SSGR and GR charts, respectively. The optimal control chart parameters are shown in Table 1. These optimal parameters are computed using the procedure described in the last two sections.

TABLE 1 : Optimal and Values of the GR Chart and SSGR Chart, Respectively

$(n, \delta_{opt}, \mathrm{ARL}_0)$	GR		SSGR	
	k	L_g	k	L_{ssg}
(3, 0.2, 370)	2.57	70	2.41	44
(3, 0.5, 370)	2.30	20	2.16	15
(3, 1.0, 370)	1.95	5	1.87	5
(5, 0.2, 370)	2.52	55	2.36	35
(5, 0.5, 370)	2.18	12	2.05	10
(5, 1.0, 370)	1.81	3	1.72	3
(7, 0.2, 370)	2.47	44	2.32	29
(7, 0.5, 370)	2.10	9	1.96	7
(7, 1.0, 370)	1.81	3	1.6	2
(3, 0.2, 500)	2.65	84	2.49	52
(3, 0.5, 500)	2.37	23	2.23	17
(3, 1.0, 500)	2.05	6	1.91	5
(5, 0.2, 500)	2.60	65	2.44	41
(5, 0.5, 500)	2.26	14	2.12	11
(5, 1.0, 500)	1.86	3	1.77	3
(7, 0.2, 500)	2.55	52	2.4	34
(7, 0.5, 500)	2.17	10	2.04	8
(7, 1.0, 500)	1.86	3	1.64	2

The optimal combinations (k, L_g) and (k, L_{ssg}) in Table 1 were used to compute the corresponding ATS(δ) for the GR and SSGR charts, which are shown in Tables 2 to 4. The ATS_G and ATS_{SSG} in Tables 2 to 4 represent the ATS values for the GR and SSGR charts, respectively. The in-control ATS for the two charts is equal to $ATS_0 = n \times ARL_0$. Table 5 shows the percentage of improvement in the ATSs of the SSGR chart as compared to the GR chart. For $\delta > 0$, the ATSs of the GR chart were larger than that of the SSGR chart, except for larger shifts where the ATSs of the GR chart were similar to that of the SSGR chart (see Tables 2 to 4). Thus, the SSGR chart had a higher detection speed of the out-of-control condition compared to the GR chart. However, the ATS_G and $ATSS_{SG}$ values were almost the same when the shifts were large ($\delta \geq 2.0$), which show that the GR and SSGR charts gave equal performance for large shifts. Table 5 shows that the percentage of improvement was larger for smaller shifts, especially for $\delta \leq 1.0$. In general, it is concluded that the SSGR chart is more sensitive than the GR chart in detecting process changes.

TABLE 2 : ATS Values of the GR and SSGR Charts when n = 3

δ	(n, δ_{opt}, ARL_0)											
	(3, 0.2, 370)		(3, 0.5, 370)		(3, 1.0, 370)		(3, 0.2, 500)		(3, 0.5, 500)		(3, 1.0, 500)	
	ATS_G	ATS_{SSG}	ATS_G	ATS_{SSG}	ATS_G	ATS_{SSG}	ATS_G	ATS_{SSG}	ATS_G	ATS_{SSG}	ATS_G	ATS_{SSG}
0.2	479.27	375.81	502.00	409.84	561.05	471.60	601.64	480.44	621.68	508.53	764.39	575.81
0.4	124.95	93.45	117.58	90.61	146.37	107.01	149.98	113.01	139.20	106.77	180.04	127.92
0.6	48.92	36.88	36.76	29.53	40.35	31.63	57.12	42.77	40.86	33.05	45.53	35.70
0.8	25.48	19.67	17.12	14.45	15.72	13.26	29.21	22.38	19.14	15.80	17.37	14.50
1.0	14.96	12.03	10.64	9.06	8.53	7.58	16.63	13.40	11.53	9.81	9.19	8.01
1.5	5.89	5.24	4.85	4.49	4.04	3.91	6.32	5.55	5.09	4.67	4.23	3.98
2.0	3.67	3.50	3.41	3.31	3.21	3.18	3.78	3.58	3.47	3.36	3.26	3.20
2.5	3.13	3.08	3.06	3.04	3.03	3.02	3.15	3.10	3.07	3.05	3.03	3.02
3.0	3.01	3.01	3.00	3.00	3.00	3.00	3.02	3.01	3.01	3.00	3.00	3.00

TABLE 3 : ATS Values of the GR and SSGR Charts when n = 5

δ	(n, δ_{opt}, ARL_0)											
	(5, 0.2, 370)		(5, 0.5, 370)		(5, 1.0, 370)		(5, 0.2, 500)		(5, 0.5, 500)		(5, 1.0, 500)	
	ATS_G	ATS_{SSG}	ATS_G	ATS_{SSG}	ATS_G	ATS_{SSG}	ATS_G	ATS_{SSG}	ATS_G	ATS_{SSG}	ATS_G	ATS_{SSG}
0.2	514.76	403.36	582.71	436.04	712.84	549.33	653.45	502.50	725.79	541.13	938.65	703.48
0.4	106.60	80.30	94.37	73.02	131.99	96.40	126.32	94.36	109.12	84.29	160.41	115.73
0.6	41.74	32.60	28.66	23.87	32.99	26.80	48.10	36.98	31.61	26.48	37.59	30.12
0.8	21.57	17.51	14.45	12.73	13.35	11.83	24.11	19.38	15.70	13.64	14.45	12.60
1.0	12.85	11.09	9.59	8.76	8.15	7.60	13.91	11.90	10.20	9.21	8.50	7.91
1.5	6.25	5.95	5.71	5.56	5.35	5.30	6.44	6.10	5.79	5.64	5.39	5.33
2.0	5.13	5.08	5.05	5.03	5.01	5.01	5.16	5.10	5.06	5.04	5.02	5.01
2.5	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00
3.0	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00

TABLE 4 : ATS Values of the GR and SSGR Charts when n = 7

δ	$(n,\delta_{opt},\mathrm{ARL}_0)$											
	(7, 0.2, 370)		(7, 0.5, 370)		(7, 1.0, 370)		(7, 0.5, 500)		(7, 1.0, 500)		(7, 1.0, 500)	
	$\mathrm{ATS_G}$	$\mathrm{ATS_{SSG}}$	$\mathrm{ATS_G}$	$\mathrm{ATS_{SSG}}$	$\mathrm{ATS_G}$	$\mathrm{ATS_{SSG}}$	$\mathrm{ATS_G}$	$\mathrm{ATS_{SSG}}$	$\mathrm{ATS_G}$	$\mathrm{ATS_{SSG}}$	$\mathrm{ATS_G}$	$\mathrm{ATS_{SSG}}$
0.2	502.38	405.46	578.10	460.36	740.46	609.92	629.02	494.90	717.41	561.10	971.45	729.34
0.4	93.10	72.69	80.16	65.40	103.67	91.44	107.79	82.93	90.59	74.66	123.86	103.67
0.6	37.44	30.34	24.98	21.87	26.64	25.54	41.88	33.74	27.00	23.68	29.50	27.54
0.8	19.35	16.67	13.82	12.50	12.47	12.08	21.14	17.99	14.65	13.22	13.12	12.53
1.0	12.27	11.09	9.90	9.33	8.98	8.62	12.97	11.66	10.24	9.66	9.17	8.79
1.5	7.51	7.35	7.22	7.15	7.10	7.06	7.60	7.43	7.25	7.19	7.11	7.07
2.0	7.01	7.01	7.00	7.00	7.00	7.00	7.02	7.01	7.00	7.00	7.00	7.00
2.5	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
3.0	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00

TABLE 5 : Percentage of Improvement of the SSGR Chart Compared to the GR Chart for $n \in \{3, 5, 7\}$

δ	$(\delta_{opt},\mathrm{ARL}_0)$																	
	(0.2, 370)			(0.5, 370)			(1.0, 370)			(0.2, 500)			(0.5, 500)			(1.0, 500)		
	n = 3	n = 5	n = 7	n = 3	n = 5	n = 7	n = 3	n = 5	n = 7	n = 3	n = 5	n = 7	n = 3	n = 5	n = 7	n = 3	n = 5	n = 7
0.2	21.59%	21.64%	19.29%	18.36%	25.17%	20.37%	15.94%	22.94%	17.63%	20.14%	23.10%	21.32%	18.20%	25.44%	21.79%	24.67%	25.05%	24.92%
0.4	25.21%	24.67%	21.92%	22.94%	22.62%	18.41%	26.89%	26.96%	11.80%	24.65%	25.30%	23.06%	23.30%	22.75%	17.58%	28.95%	27.85%	16.30%
0.6	24.61%	21.90%	18.96%	19.67%	16.71%	12.45%	21.61%	18.76%	4.13%	25.12%	23.12%	19.44%	19.11%	16.23%	12.30%	21.59%	19.87%	6.64%
0.8	22.80%	18.82%	13.85%	15.60%	11.90%	9.55%	15.65%	11.39%	3.13%	23.38%	19.62%	14.90%	17.45%	13.12%	9.76%	16.52%	12.80%	4.50%
1.0	19.59%	13.70%	9.62%	14.85%	8.65%	5.76%	11.14%	6.75%	4.01%	19.42%	14.45%	10.10%	14.92%	9.71%	5.66%	12.84%	6.94%	4.14%
1.5	11.04%	4.80%	2.13%	7.42%	2.63%	0.97%	3.22%	0.93%	0.56%	12.18%	5.28%	2.24%	8.25%	2.59%	0.83%	5.91%	1.11%	0.56%
2.0	4.63%	0.97%	0.00%	2.93%	0.40%	0.00%	0.93%	0.00%	0.00%	5.29%	1.16%	0.14%	3.17%	0.40%	0.00%	1.84%	0.20%	0.00%
2.5	1.60%	0.00%	0.00%	0.65%	0.00%	0.00%	0.33%	0.00%	0.00%	1.59%	0.00%	0.00%	0.65%	0.00%	0.00%	0.33%	0.00%	0.00%
3.0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.33%	0.00%	0.00%	0.33%	0.00%	0.00%	0.00%	0.00%	0.00%

AN ILLUSTRATIVE EXAMPLE

To show the implementation and application of the GR and SSGR control charts, a set of real data from Wild and Seber (2000) was adopted. Consider a dry powder filling process, where the weight of content in each can is the quality characteristic of interest. In a canning plant, dry powder is packed into cans with a nominal weight of 2000 grams. Cans are filled at a four-head filler fed by a hopper, with each head filling about 25 cans every two minutes.

The process is controlled by a computer programme that calculates very crude adjustments to fill-times based on the filled weight recorded at a check weigher. A sub-group of 5 cans is a natural sub-group size for studying the process. The powder density is expected to change often. This is because of the settling effect in the bins where it was stored prior to canning. Therefore, automatic adjustment is needed.

The weights were recorded for 5 successive cans every two minutes, for 60 minutes, to see how the process was performing. Hence, the data consisted of 30 sub-groups, each of size $n = 5$. These weights, expressed as deviations from 1984 grams, are given in Table 6. The mean $\bar{X}_i$ and range R_i of each of these subgroups are also given in Table 6.

TABLE 6 : Canning Plant Data

Subgroup	Weight-1984 (grams)					Mean	Range
1	32.3	31.6	13.3	14.3	16.6	21.62	19.0
2	23.2	32.9	30.1	34.8	29.9	30.18	11.6
3	8.1	17.5	11.9	11.4	12.5	12.28	9.4
4	19.6	26.2	27.8	27.4	17.1	23.62	10.7
5	31.4	35.7	29.2	29.7	26.9	30.58	8.8
6	37.5	22.6	8.1	12.9	14.5	19.12	29.4
7	20.0	18.0	23.6	9.0	16.1	17.34	14.6
8	7.9	4.4	4.4	3.9	3.7	4.86	4.2
9	17.8	17.1	18.4	24.9	21.5	19.94	7.8
10	25.4	26.9	27.3	21.6	29.2	26.08	7.6
11	35.9	42.8	41.1	37.4	24.8	36.40	18.0
12	26.6	33.4	27.9	25.1	29.9	28.58	8.3
13	13.7	11.8	20.6	6.2	14.2	13.30	14.4
14	32.3	23.1	17.7	22.1	12.1	21.46	20.2
15	27.4	26.0	29.4	29.5	32.5	28.96	6.5
16	36.5	42.4	30.7	27.0	23.3	31.98	19.1
17	24.0	36.8	31.5	22.5	25.6	28.08	14.3
18	26.2	18.0	14.4	6.8	11.3	15.34	19.4
19	25.7	26.3	23.2	17.8	18.1	22.22	8.5
20	16.4	44.1	33.4	29.7	32.2	31.16	27.7
21	13.2	23.3	23.7	21.0	16.7	19.58	10.5
22	24.5	32.8	24.4	29.2	22.0	26.58	10.8
23	16.7	24.9	27.8	29.3	31.4	26.02	14.7

TABLE 6 : (Continued)

24	34.2	25.6	11.5	8.5	2.6	16.48	31.6
25	33.6	17.4	17.5	18.4	15.6	20.50	18.0
26	27.2	37.2	27.4	28.2	21.2	28.24	16.0
27	29.6	39.0	35.7	32.5	29.3	33.22	9.7
28	18.9	54.3	40.4	35.3	28.3	35.44	35.4
29	19.1	28.6	23.8	29.9	27.1	25.70	10.8
30	29.9	29.4	30.8	30.3	38.5	31.78	9.1

Firstly, the optimal values of (k, L_g) for the GR chart and (k, L_{ssg}) for the SSGR chart were obtained based on (n = 5, δ_{opt} = 1, ATS_0 = 2000). We used the optimal values $(k, L_g) = (1.82, 3)$, as given in Gadre and Rattihalli (2004) for the GR chart, while optimal values of (k, L_{SSG}) = (1.74, 3) as given in Gadre and Rattihalli (2007), were adopted for the SSGR chart.

Next, the $\bar{X}$ sub chart was set up to declare whether a sub-group of data was conforming or non-conforming. The centre line was computed as $\hat{\mu}_0 = \frac{\sum_{i=1}^{m} \bar{X}_i}{m} = 24.22$, where m = 30 is the number of subgroups. Since there were five observations in each subgroup i.e. n = 5, the range method was used to estimate the standard deviation σ for this $\bar{X}$ sub chart. The average sample range was calculated as $\bar{R} = \frac{\sum_{i=1}^{m} R_i}{m} = 14.9$. Thus, the standard deviation for the $\bar{X}$ sub chart was estimated as $\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{14.9}{2.326} = 6.41$.

Therefore, with $\hat{\mu}_0 = 24.22$ and $\hat{\sigma} = 6.41$, the lower and upper control limits of the $\bar{X}$ sub chart could be computed. For the GR chart, the lower and upper control limits of the sub chart were $L_{\bar{X}|S} = 19.0027$ and $U_{\bar{X}|S} = 29.4373$, while for the SSGR chart, the lower and upper control limits of the $\bar{X}$ sub chart were $L_{\bar{X}|S}$ = 19.232 and $U_{\bar{X}|S}$ = 29.208 . Hence, the $\bar{X}$ sub chart for the weights of dry powder in cans could be constructed as given in Fig.1 and Fig.2 for the GR and SSGR charts, respectively.

The dots in Fig.1 and Fig.2 are the non-conforming groups of canning plant data as those group means fell outside the lower and upper limits of the $\bar{X}$ sub chart. Let Y_r be the *rth* group-based CRL, for r ∈ { 1, 2, 3,.. } . Recall that the GR chart indicates an out-of-control signal if either $Y_1 \le L_g$ or two successive Y_r and Y_{r+1} (for r = 2, 3, …) are less than or equal to L_g for the first time, while for the SSGR chart, a process indicates an out-of-control signal if either $Y_1 \le L_{ssg}$ or two successive Y_r and Y_{r+1} (for r = 2, 3, …) are less than or equal to L_{ssg} for the first time, provided that the two successive $\bar{X}$ samples corresponding to the two successive group-based CRLs fall on the same side of the target value μ_0. From Fig.1 and Fig.2, we know that $Y_1 = 2$, which is less than $L_G = L_{SSG} = 3$. Therefore, the process was declared as being out-of-control by both the GR and SSGR control charts.

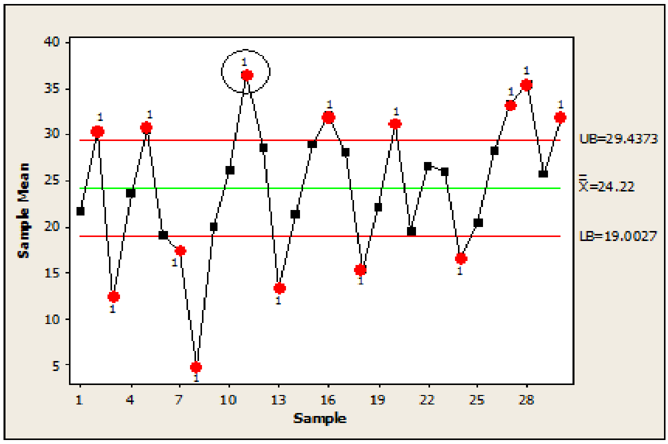

Fig.1: $\bar{X}$ sub chart for the weights of dry powder in cans (GR chart).

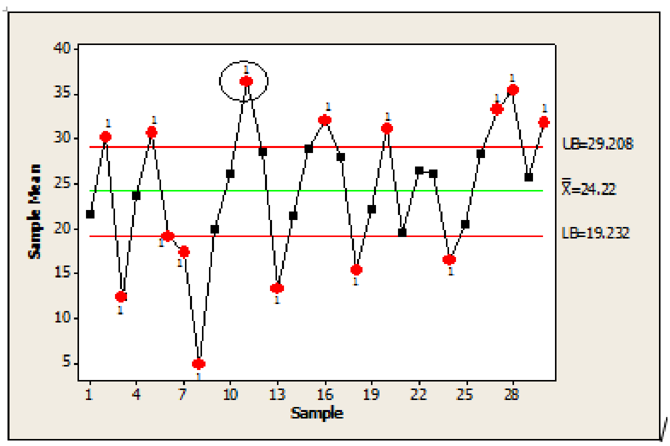

Fig.2: $\bar{X}$ sub chart for the weights of dry powder in cans (SSGR chart).

However, if we assume that all of the subgroups before the 11th (in circle) subgroup were conforming groups i.e. the first 10 subgroups have points plotting within the $L_{\overline{X}|S}$ and $U_{\overline{X}|S}$ limits, then $Y_1 = 11$, $Y_2 = 2$ and $Y_3 = 3$. Thus, for the GR chart, the process was declared as being out-of-control at the 16th sub-group as both Y_2 and Y_3 were less than or equal to L_g. However, the SSGR chart would not declare the process as being out-of-control after observing the third non-conforming group although both Y_2 and Y_3 were less than or equal to L_{ssg} . This is because the group means for Y_2 and Y_3 fell on the opposite side of the target value $\bar{X}$. The SSGR chart would only declare the dry powder filling process as out-of-control after plotting Y_8, as both $Y_7 = 3$ and $Y_8 = 1$ are not greater than L_{SSG} and both group means fall on the same side of the $\bar{X}$ sub chart. Thus, when the SSGR chart was adopted, the process was declared as being out-of-control at the 28th sub-group.

CONCLUSION

This study assumed that the underlying distribution followed an independent and identically distributed normal distribution. In this study, the ATSs of the GR and SSGR charts were compared for different sizes of mean shifts. When the in-control ATS of the charts under comparison was fixed at the same value, the chart having the smallest out-of-control ATS among all the competing charts was preferable. Since the ATS values of the SSGR chart were significantly less than the ATS values of the GR chart, the SSGR chart surpassed the GR chart for any sizes of shifts in the process mean.

Future research can be done to compare the performance of the GR and SSGR charts with estimated process parameters. Besides that, a comparative study of the performance of the GR and SSGR charts for skewed and heavy-tailed distributions can also be explored in future research.

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